# TRIGONOMETRY

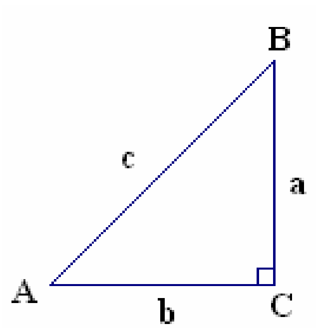
## *FOR RIGHT TRIANGLES*

***Written by Professor Barbara Maurice***



▲ THINGS TO KNOW BEFORE YOU START

- THE PYTHAGOREAN THEOREM – APPLIES ONLY TO RIGHT TRIANGLES




► C is the right angle and measures 90°

► Angles A and B are acute angles.(Acute angles measure less than 90° They are complementary angles and have a sum of 90°

► c is the hypotenuse and directly opposite Angle C – the right angle

► a and b are the legs and directly opposite their respective angles

$$a^2 + b^2 = c^2$$

EXAMPLE 1:

A right triangle has a leg whose length is 6 cm and the hypotenuse is 10 cm. Find the length of the other leg.

$a^2 + b^2 = c^2$

$6^2 + b^2 = 10^2$

$36 + b^2 = 100$ $b^2 = 64$ $b = 8$ The leg is 8 cm.

EXAMPLE 2:

An isosceles right triangle has legs whose lengths are 5 inches. Find the length of the hypotenuse.

$a^2 + b^2 = c^2$

$5^2 + 5^2 = c^2$

$25 + 25 = c^2$

$50 = c^2$ or $c = \sqrt{50}$; $c = \sqrt{25 * 2} = 5\sqrt{2}$ ; The hypotenuse is $5\sqrt{2} \approx 7.1$ inches.

- ROUNDING

  Round lengths to one place more than the given information
  Round trig values to 4 decimal places.

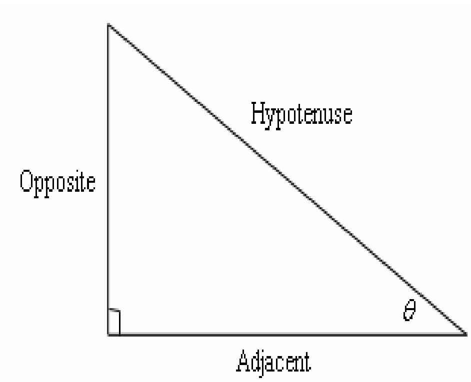
- FREQUENTLY USED GREEK LETTERS TO DESIGNATE ANGLES

  $\alpha$ - alpha $\beta$ - beta $\theta$ - theta

**<u>THE SIX TRIGONOMETRIC FUNCTIONS</u>**

sine of angle θ: $\sin\theta = \dfrac{\textit{opposite leg}}{\textit{hypotenuse}}$

cosine of angle θ: $\cos\theta = \dfrac{\textit{adjacent leg}}{\textit{hypotenuse}}$




tangent of angle θ: $\tan\theta = \dfrac{\textit{opposite leg}}{\textit{adjacent leg}}$

cosecant of angle θ: $\csc\theta = \dfrac{\textit{hypotenuse}}{\textit{opposite leg}}$

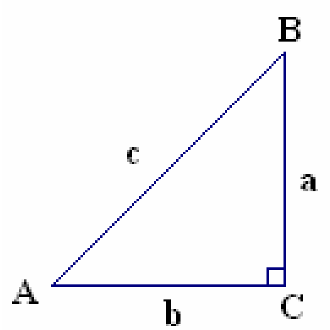
secant of angle θ: $\sec\theta = \dfrac{\textit{hypotenuse}}{\textit{adjacent leg}}$

cotangent of angle θ: $\cot\theta = \dfrac{\textit{adjacent leg}}{\textit{opposite leg}}$

## ► *SohCahToa* ◄

- Note – the cosecant, secant, and cotangent are the reciprocal functions of the sine, cosine, and tangent respectively.

The following shows the trigonometric ratios for acute angles A and B.




$\sin A = \dfrac{a}{c}$ $\csc A = \dfrac{c}{a}$

$\cos A = \dfrac{b}{c}$ $\sec A = \dfrac{c}{b}$

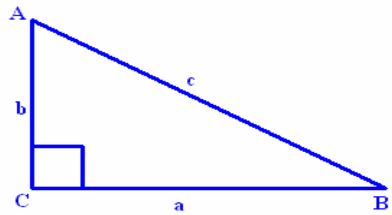
$\tan A = \dfrac{a}{b}$ $\cot A = \dfrac{b}{a}$

$\sin B = \dfrac{b}{c}$ $\csc B = \dfrac{c}{b}$

$\cos B = \dfrac{a}{c}$ $\sec B = \dfrac{c}{a}$

$\tan B = \dfrac{b}{a}$ $\cot B = \dfrac{a}{b}$

For examples 3 and 4, use the triangle below to find the six trigonometric functions.




EXAMPLE 3: $a = 3, b = 4, c = 5$ (use acute angle B)

$$\sin B = \frac{4}{5} \qquad \csc B = \frac{5}{4}$$

$$\cos B = \frac{3}{5} \qquad \sec B = \frac{5}{3}$$

$$\tan B = \frac{4}{3} \qquad \cot B = \frac{3}{4}$$

EXAMPLE 4: $a = 8, c = 12$ (Use acute angle A)

Use the Pythagorean Theorem to find b:

$a^2 + b^2 = c^2$

$8^2 + b^2 = 12^2$

$b^2 = \sqrt{12^2 - 8^2}$

$b = \sqrt{80} = 4\sqrt{5}$ (In this problem do not rationalize denominators)

$$\sin A = \frac{8}{12} = \frac{2}{3} \qquad \csc A = \frac{3}{2}$$

$$\cos A = \frac{4\sqrt{5}}{12} = \frac{\sqrt{5}}{3} \qquad \sec A = \frac{3}{\sqrt{5}}$$

$$\tan A = \frac{8}{4\sqrt{5}} = \frac{2}{\sqrt{5}} \qquad \cot A = \frac{\sqrt{5}}{2}$$

- THE CALCULATOR – MAKE SURE THAT YOUR CALCULATOR IS IN DEGREE MODE

FIND: $\sin 37°$ — $\sin 37° \approx .6018$

$\cos 58°$ — $\cos 58° \approx .5299$

$\tan 17°$ — $\tan 17° \approx .3057$

FIND θ: $\sin\theta = .7986$ — [ $\sin^{-1}$ .7986 enter] ; $\theta \approx 53°$

$\cos\theta = .9397$ — [ $\cos^{-1}$ .9397 enter] ; $\theta \approx 20°$

$\tan\theta = 1.8807$ — [$\tan^{-1}$ 1.8807 enter] ; $\theta \approx 62°$

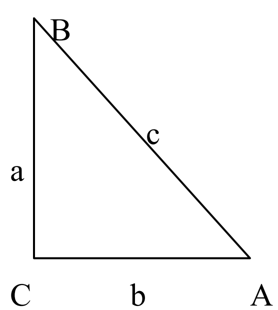
- SOLVING RIGHT TRIANGLES – FINDING THE MEASURES OF THE TWO ACUTE ANGLES AND THE THREE SIDES OF THE TRIANGLE

- You can find the measure of each acute angle of a right triangle and the length of all three sides if you know:

1. the length of a side and the measure of an acute angle or
2. the lengths of two sides

■ Note: Use only given information to find unknown values.

EXAMPLE 5: Given: b = 6 inches; A = 24º (C is a right angle)
Find: a, c, B




<u>To find a</u>: $\tan A = \frac{opp}{adj}$ $\qquad \tan 24° = \frac{a}{6}$

$a = 6(\tan 24°) \approx 2.7$ inches

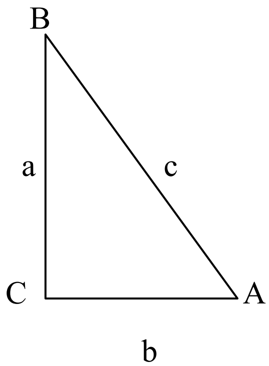
<u>To find c</u>: $\cos A = \frac{adj}{hyp}$ $\qquad \cos 24° = \frac{6}{c}$

$c\cos 24° = 6 \qquad c = \frac{6}{\cos 24°} \qquad c \approx 6.6$ inches

<u>To find B</u>: $90° - 24° = 66°$

EXAMPLE 6: Given: a = 22cm; c = 34cm (C is a right angle)

Find: b, A, B




To find b: $a^2 + b^2 = c^2$

$22^2 + b^2 = 34^2$

$b = \sqrt{34^2 - 22^2}$ $\quad b \approx 25.9$ cm

To find A: $\sin A = \frac{opp}{hyp}$

$\sin A = \frac{22}{34}$ $\quad [\sin^{-1} \frac{22}{34}$ enter]

$A \approx 40.3°$

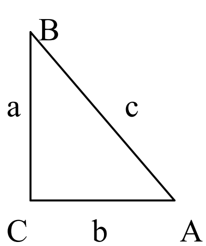
To find B: $\cos B = \frac{adj}{hyp}$

$\cos B = \frac{22}{34}$ $\quad [\cos^{-1} \frac{22}{34}$ enter]

$B \approx 49.7°$

EXAMPLE 7: Given: A = 27º ; a = 6.8 mm (C is a right angle)

Find B, b, c




To find B: 90º – 27º = 63º

To find b: $\tan A = \frac{opp}{adj}$ $\quad \tan 27° = \frac{6.8}{b}$

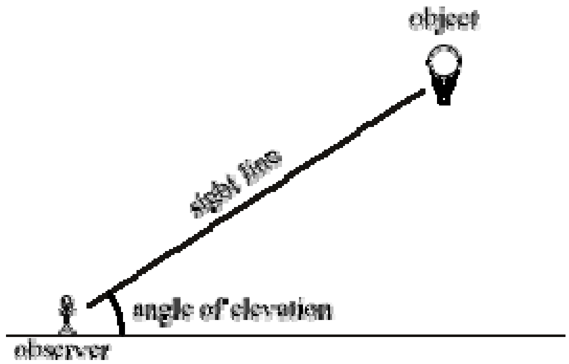
$b \tan 27° = 6.8$ $\quad b = \frac{6.8}{\tan 27°} \approx 13.35$ mm

To find c: $\sin A = \frac{opp}{hyp}$ $\quad \sin 27° = \frac{6.8}{c}$

$c \sin 27° = 6.8$ $\quad c = \frac{6.8}{\sin 27°}$ $\quad c \approx 14.98$ mm

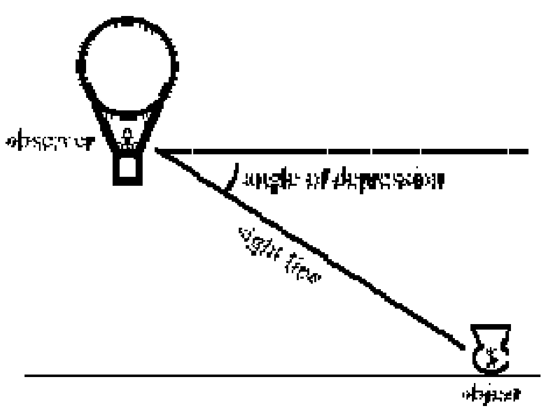
ANGLE OF ELEVATION

The angle of elevation refers to the angle between a horizontal line and the line of sight to an object, when the object being sighted is above the observer.




ANGLE OF DEPRESSION

The angle of depression refers to the angle between a horizontal line and the line of sight to an object, when the object being sighted is below the observer.




- APPLICATIONS OF TRIGONOMETRY AND HOW TO APPROACH THEM

  Read the problem carefully

  Draw a picture

  Label the given information

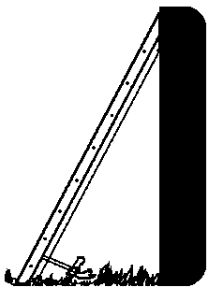
  Identify the unknown and label it

  Choose the appropriate trigonometric identity and solve

  Check that your answer is sensible

EXAMPLE 8:

A sixteen foot ladder leans against a building and makes a 65° angle with the ground. How far is the ladder from the base of the building?



Solution

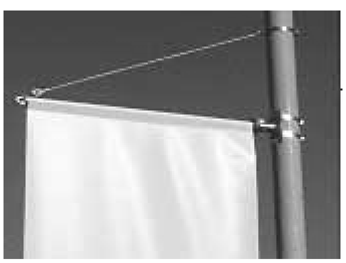
$\cos 65° = \dfrac{x}{16}$ $\quad 16 \cos 65° = x$

$x \approx 6.8$

*The ladder is approximately 6.8 feet from the base of the building.*

EXAMPLE 9:

A thirteen-foot wide banner is hung on a horizontal rod attached to a flagpole. The rod is supported by a wire that makes a 39° angle with the pole. How long must the supporing wire be?



*Solution*

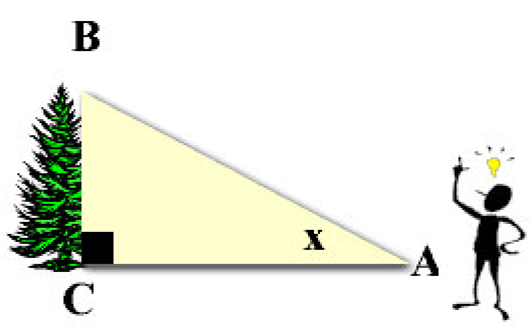
$\sin 39° = \dfrac{13}{x}$ $\quad x \sin 39° = 13$

$x = \dfrac{13}{\sin 39°} \approx 20.7$

*The supporting wire must be approximately 20.7 feet.*

EXAMPLE 10:

A forty-three foot tree casts a 60 foot shadow. Find the angle of elevation of the sun.




*Soloution*

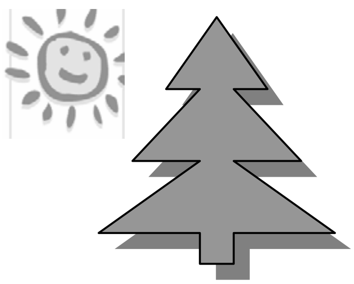
$\tan A = \dfrac{43}{60}$ $\quad A = \tan^{-1} \dfrac{43}{60}$

A ≈ 35.6

The angle of elevation of the sun is approximately 35.6°

EXAMPLE 11:

The angle of elevation to the top of a forty-three foot tree is 62°. Find the length of the tree's shadow.



*Solution*

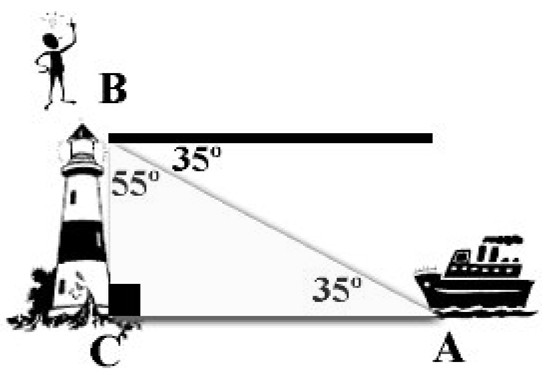
$tan\ 62° = \dfrac{43}{x}$ $\quad x\ tan\ 62° = 43$

$x = \dfrac{43}{\tan 62°}$ $\quad x \approx 22.9$

*The tree's shadow is approximately 22.9 feet*

EXAMPLE 12:

The angle of depression from the top of a lighthouse to a cruise ship is 35°. If the height of the lighthouse is 84 feet, how far is the cruise ship from the base of the lighthouse?




*Solution*

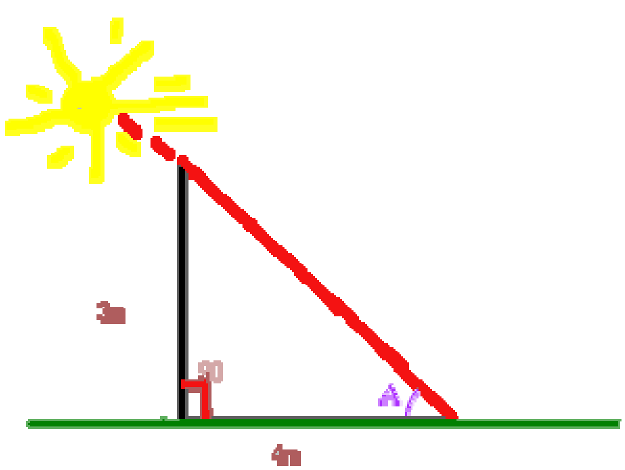
$tan\ 55° = \dfrac{x}{84}$ $\quad 84\ tan\ 55° = x$

$x = 120.0$

*The ship is approximately 120.0 feet from the base of the lighthouse.*

EXAMPLE 13:

At a certain time of day, a vertical pole 3 meters tall casts a 4 meter shadow. Find the angle of elevation of the sun.




*Solution*

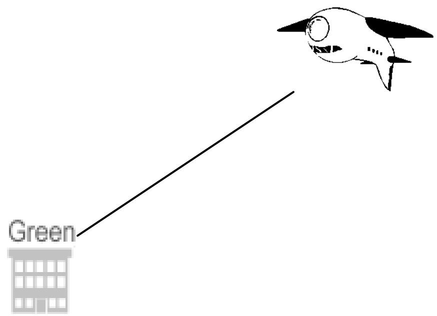
$tan\ A = \dfrac{3}{4}$ $\quad A = tan^{-1}\ \dfrac{3}{4}$

$A \approx 36.9°$

*The angle of elevation of the sun Is approximately 36.9°*

EXAMPLE 14:

An airplane is flying at an altitude of 4800 feet. The pilot is heading for Green Airport. The angle of depression to the airport is 48°. What is the flying distance between the plane and Green Airport?




*Solution*

$\cos 42° = \dfrac{4800}{x}$ $\quad x \cos 42° = 4800$

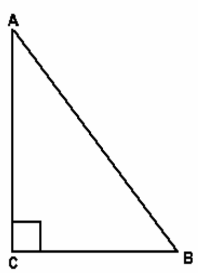
$x = \dfrac{4800}{\cos 42°}$ $\quad x \approx 6459.0$

*The flying distance to the airport is approximately 6459.0 feet.*

**PRACTICE EXERCISES**

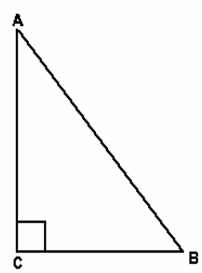
1. – 4. Find the exact values of the six trigonometric functions of each acute angle in the given right triangles. (See examples 3 and 4)

1.



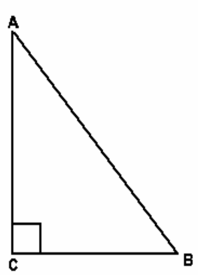

a = 6
b = 8
c = 10

2.



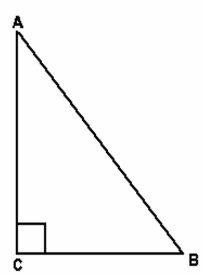

b = 5
a = 8

3.




a = 7
c = 11

4.




b = x
c = 2x

5. – 8. Use a calculator to approximate each of the following to 4 decimal places.

5. cos 9°  6. tan 22°  7. sin 68.4°  8. cos 83.2°

9. – 12. Use a calculator to approximate the measure of angle θ to the nearest tenth of a degree.

9. sin θ ≈ .6868  10. tan θ ≈ .4273  11. cos θ ≈ .3417  12. tan θ ≈ .0489

13. – 20. Solve each right triangle (C = 90°) for the given data. (See examples 5, 6, and 7)

13. A = 27°, a = 6.8 ft

14. b = 5 cm, c = 12 cm

15. B = 36 °, a = 9 m

16. a = 16 in, b = 9 in

17. A = 68.4°, c = 40.8 ft

18. a = 6 m, c = 20 m

19. B = 42°, b = 18 in

20. A = 26.7°, b = 58 ft

21. – 31. Solve each of the following problems, following the steps on page 5. (See examples 8 – 14)

21. A television transmission tower, which is 128 meters high, is supported by a cable that is attached to the top of the tower and anchored to the ground. The cable forms a 72.4° angle with the ground. Find the distance between the cable anchor and the base of the tower. How long must the cable be?

22. From JFK Airport, the angle of elevation of a plane is 23°. If the plane's altitude is 2500 meters what is its flying distance to the airport?

23. A sixty-two foot tree casts a 41 foot shadow. What is the angle of elevation from the end of the shadow to the top of the tree?

24. Sal is an avid birdwatcher and spots a crow sitting on top of a lamppost. The angle of depression from the crow to Sal is 35°. If the distance between Sal and the crow (as the crow flies) is 25 feet, find the height of the lamppost.

25. Jan wants to measure the height of a tree. She walks 100 feet from the base of the tree and looks at the tree's top. The angle from the ground to the top of the tree is 33°. How tall is the tree?

26. Pete the painter uses a twenty foot ladder, which he leans against a house forming a 65° angle with the ground. How far up the house can Peter paint?

27. An airplane is flying at an altitude of 2.5 miles. The distance along the ground, from a point directly below the airplane to the airport, is 14.6 miles. What is the angle of depression from the airplane to the airport?

28. The legs of an isosceles triangle are 8 inches long. If the base angles are 35°, find the length of the third side of the triangle.

29. At 2 p.m. on a sunny afternoon, a building which is 528 feet tall casts a shadow that is 136 feet long. What is the angle of elevation of the sun?

30. A fifty foot ramp in a parking garage makes a 15° angle between one floor and the one above it. What is the vertical distance between the two floors?

31. A lighthouse keeper observes a ship in the ocean. The angle of depression from the top of the lighthouse, which is 85 feet above sea level, to the ship is 3.6°. How far is the ship from the base of the lighthouse?

<u>ANSWERS TO PRACTICE EXERCISES</u>

1. $\sin A = \frac{3}{5}$ $\quad \sin B = \frac{4}{5}$

$\cos A = \frac{4}{5}$ $\quad \cos B = \frac{3}{5}$

$\tan A = \frac{3}{4}$ $\quad \tan B = \frac{4}{3}$

$\csc A = \frac{5}{3}$ $\quad \csc B = \frac{5}{4}$

2. $\sin A = \frac{8}{\sqrt{89}}$ $\quad \sin B = \frac{5}{\sqrt{89}}$

$\cos A = \frac{5}{\sqrt{89}}$ $\quad \cos B = \frac{8}{\sqrt{89}}$

$\tan A = \frac{8}{5}$ $\quad \tan B = \frac{5}{8}$

$\csc A = \frac{\sqrt{89}}{8}$ $\quad \csc B = \frac{\sqrt{89}}{5}$

$\sec A = \frac{5}{4}$ $\sec B = \frac{5}{3}$ $\sec A = \frac{\sqrt{89}}{5}$ $\sec B = \frac{\sqrt{89}}{8}$

$\cot A = \frac{4}{3}$ $\cot B = \frac{3}{4}$ $\cot A = \frac{5}{8}$ $\cot B = \frac{8}{5}$

3. $\sin A = \frac{7}{11}$ $\sin B = \frac{6\sqrt{2}}{11}$

$\cos A = \frac{6\sqrt{2}}{11}$ $\cos B = \frac{7}{11}$

$\tan A = \frac{7}{6\sqrt{2}}$ $\tan B = \frac{6\sqrt{2}}{7}$

$\csc A = \frac{11}{7}$ $\csc B = \frac{11}{6\sqrt{2}}$

$\sec A = \frac{11}{6\sqrt{2}}$ $\sec B = \frac{11}{7}$

$\cot A = \frac{6\sqrt{2}}{7}$ $\cot B = \frac{7}{6\sqrt{2}}$

4. $\sin A = \frac{\sqrt{3}}{2}$ $\sin B = \frac{1}{2}$

$\cos A = \frac{1}{2}$ $\cos B = \frac{\sqrt{3}}{2}$

$\tan A = \sqrt{3}$ $\tan B = \frac{1}{\sqrt{3}}$

$\csc A = \frac{2}{\sqrt{3}}$ $\csc B = 2$

$\sec A = 2$ $\sec B = \frac{2}{\sqrt{3}}$

$\cot A = \frac{1}{\sqrt{3}}$ $\cot B = \sqrt{3}$

5. 0.9877 6. 0.4040 7. 0.9298 8. 0.1184

9. 43.4° 10. 23.1° 11. 70.0° 12. 2.8°

13. B = 63°, b = 13.35 ft, c = 14.98 ft

14. A = 65.4°, B = 24.6°, a = 10.9 cm

15. A = 54°, b = 6.5 m, c = 11.1 m

16. A = 60.6°, B = 29.4°, c = 18.4 in

17. B = 21.6°, a = 37.93 ft, b = 15.02 ft

18. A = 17.5°, B = 72.5°, b = 19.1 m

19. A = 48°, a = 20.0 in, c = 26.9 in

20. B = 63.3°, a = 29.2 ft, c = 64.9 ft

21. 40.6 meters from the base of the tower; cable must be 134.3 meters

22. 6398.3 meters

23. 56.5°

24. 14.3 feet

25. 64.9 feet

26. 18.1 feet

27. 9.7°

28. 13.1 in

29. 75.6°

30. 12.9 feet

31. 1351.0 feet